

## B.Sc. Semester - 4 (NEP-2020) Examination

March/April-2025

## MATH: Linear Algebra-2 (Major-8)

Time: 2:00 Hours

Marks: 50

## Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

- Que-1 Attempt any two out of three (10)
1. If  $T$  is a symmetric linear map on a real inner product space  $V$  then prove that orthogonal complement subspace of  $N_T$  is  $R_T$ .
  2. State and prove first form of spectral theorem.
  3. Find diagonal matrix which is congruent to  $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ .
- Que-2 Attempt any two out of three (10)
1. If  $\varphi$  is a bilinear form on  $\mathbb{R}^n$  then there exists a unique matrix  $A_{n \times n}$  such that  $\varphi(x, y) = xAy^T, x, y \in \mathbb{R}^n$ .
  2. Check whether  $\varphi((x_1, x_2, x_3), (y_1, y_2, y_3)) = x_1y_1 + 2x_1y_2 - x_1y_3 - x_2y_1 + x_2y_2 + x_3y_1 + x_3y_3$  is a bilinear form on  $\mathbb{R}^3$  or not.
  3. Find the matrix of the bilinear form  $\varphi(x, y) = 2x_1y_1 - 3x_1y_2 + x_2y_2$  with respect to the basis  $\{(1, 0), (1, 1)\}$ .
- Que-3 Attempt any two out of three (10)
1. If  $Q(x)$  is a homogeneous quadratic form on  $\mathbb{R}^n$  then there exists a unique matrix  $A_{n \times n}$  such that  $Q(x) = xAx^T, \forall x \in \mathbb{R}^n$ .
  2. Find power, signature, rank, reduced form and canonical form of homogeneous quadratic form  $Q(x) = 2x_1^2 + 10x_1x_2 + 4x_2^2$  if possible.
  3. Check whether matrices  $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$  and  $\begin{pmatrix} 1 & 14 \\ 8 & 67 \end{pmatrix}$  are congruent or not.
- Que-4 Attempt any two out of three (10)
1. The set of all translation linear map on vector space  $V$  with composition binary operation is a commutative group.
  2. Composition of two affine linear maps on vector space  $V$  is also an affine linear map.
  3. Find an affine transformation  $H: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  such that  $H(1, 2) = (3, 2), H(2, 2) = (4, 1)$  and  $H(2, 3) = (7, 2)$
- Que-5 Attempt any two out of three (10)
1. Find an orthogonal complement subspace of the subspace  $S = \text{Sp}\{(1, -1, 0), (0, 2, -1)\}$  of  $\mathbb{R}^3$ .
  2. If  $\varphi$  is a bilinear form on  $V$  and if  $\varphi(x, y) = \Psi(y, x), \forall x, y \in V$  then check whether  $\Psi$  is a bilinear form on  $V$  or not.
  3. Check whether  $\begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \end{pmatrix}$  is an orthogonal matrix or not. If possible, find its inverse.

\*\*\*\*\*